The Effects of Crack Size on Crack Identification in a Freely Vibrating Plate using Bayesian Parameter Estimation

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ABSTRACT

In this paper a new approach is taken to identify a crack in a simply supported plate undergoing free vibration. The approach uses a Markov-Chain Monte-Carlo implementation of Bayes' Rule to identify the presence of a crack and, more importantly, to estimate crack parameters; the process also provides confidence intervals for those parameters. To generate the required time series, a semi-analytical free response is constructed out of a finite element based eigensolution. This detection technique is applied to a cracked plate and effectively identifies the crack location, orientation and length. The results show the utility and accuracy of this method for a variety of cracks lengths, suggesting that even small cracks may be detected.

INTRODUCTION

Cracks in plates tend to undermine structural performance and shorten lifetimes. As such, a reliable and automated means for finding cracks early (while small) would be of considerable use. But beyond identifying the presence of the damage, it would be helpful to obtain information regarding the size, location, and orientation of a crack. An ideal non-destructive crack detection method would be able to: (i) find small cracks using a minimum number of sensors, (ii) work reliably in real-time, (iii) operate successfully regardless of environmental changes in the field, and (iv) provide results without response information from an undamaged (healthy) structure as a baseline.

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14. ABSTRACT
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Previous efforts have focused primarily on the presence and sometimes location of damage, often by looking at variations in the vibration modes and frequency spectra [1]. While other "features" in the data can be used, the basic idea is to look for damage-induced changes, or damage-specific patterns in the data (see e.g., [2]). The main issue with the pattern recognition approach is that it cannot provide specific information about the damage magnitude or orientation. Making such an assessment would require storing vibration data from every possible size, location, orientation, etc. and training a classifier. In addition, one must consider environmental-induced patterns (e.g. temperature fluctuations). In short, data-driven approaches can only be as good as the previously acquired data allows.

A different approach is to treat the problem as one of estimation: given a parameterized damage model, estimate the parameters associated with the damage. Recent efforts toward this goal include the work of Horibe & Watanabe [3] (using genetic algorithms) and a Bayesian identification approach used by Ng *et al.*[4]. Treating the problem as one of estimation eliminates the need for baseline data and buys the practitioner some immunity from environmental variability. The challenge instead becomes one of developing a model that captures the influence of structural damage and then developing an estimator for the parameters in that model.

Here we propose a model-based approach that uses a Markov-Chain Monte-Carlo (MCMC) implementation of Bayes' theorem to identify the damage parameters. Analytical models for a plate with a single crack exist [5], but extension to edge cracks, branched cracks or multiple cracks is, to the authors' knowledge, an unsolved problem. The finite element method (FEM) provides more flexibility in the geometry considered, for both the plate and the crack. In this work, a semi-analytical/FEM model is used. Finite elements are used to obtain the frequencies and mode shapes and an analytical solution, based on this numerical eigen-solution, is developed for the time response. The Bayesian/MCMC approach developed here is used to identify the location, orientation, and size of a crack. Additionally, because the Bayesian approach provides estimates of the entire parameter probability distribution, confidence intervals may be easily obtained. The method shows considerable promise for use in real applications.

DYNAMIC MODEL

The objective is to arrive at a computationally fast means for obtaining the time response of the cracked plate. A classical modal analysis approach is taken. To obtain the required eigenvalues and eigenvectors, the finite element method is used. The structured mesh shown in Figure 1b is typical of those used in this study. The model matches the theoretical predictions of the first and second natural frequencies for a cracked plate [5] to within 0.5%. The unknown model parameters are: the location of the center of the crack (x_{crack}, y_{crack}), the half crack length (a), and the orientation of the crack measured from the positive x-axis (α).

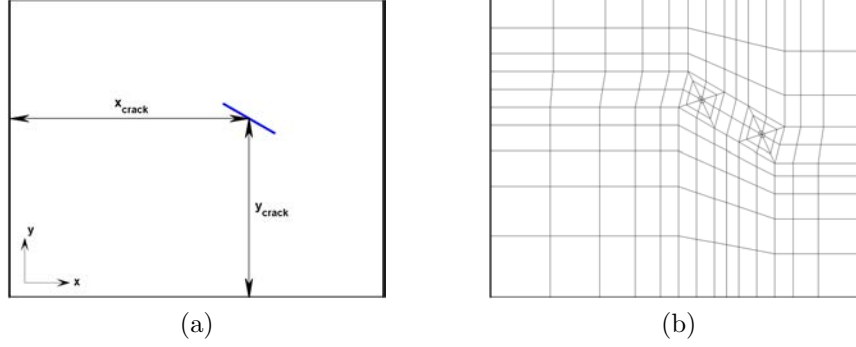Fast efficient forward models are extremely important when using Monte

Figure 1: (a) Schematic of Plate and Crack and (b) Typically Course FEM Mesh

Carlo methods such as the one proposed here. We have therefore chosen to use quadrilateral Mindlin serendipity elements in forming the model. The elements directly surrounding each crack tip are modified [6] to capture the $1/\sqrt{r}$ stress distribution that dominates in the vicinity of the crack tip. This significantly reduces the required mesh density and improves run-time. Small deflections are assumed throughout. Also, it is assumed that the crack remains open, there is no appreciable mass loss, and the crack does not grow. The crack tips are restricted to be at least a half crack length (a) from the edge of the plate.

THE BAYESIAN ESTIMATION APPROACH

The prerequisites for a Bayesian analysis are observed, experimental data ($\mathbf{y}$) from the (possibly) damaged structure and a mathematical model containing damage parameters (θ). The goal is to estimate the conditional posterior probability distribution $p(\theta_i|\mathbf{y})$, for the crack parameters, θ, given the data, $\mathbf{y}$. Here, the crack parameters are the location of the crack center, the crack length, and the crack orientation relative to the positive x-axis: $\theta = \{x_{\text{crack}}, y_{\text{crack}}, a, \alpha\}$. Any *a-priori* information about these parameters is encoded in the prior distribution, $p_\pi(\theta_i)$. The key ingredient relating these two distributions is the likelihood, $p_L(\mathbf{y}|\theta, \sigma^2)$, formed by considering the net error in our observations. The predicted response at some location "r" on the structure at time "n" will differ from that of the experimental data by some amount. At the rth sensor, this is

$$y_{rn} = x_{rn} + \eta_{rn}, \quad n = 1, 2, 3....N, \quad r = 1, \cdots, M \tag{1}$$

where η_{rn} is the net error (model error + experimental noise), N is the number of elements in the time series, and M is the number of sensors. If the error is assumed Gaussian and independent, then the probability that the data $\mathbf{y}$ was produced by the model, under the assumed parameters (θ) and variance (σ) is:

$$p_L(\mathbf{y}|\theta, \sigma^2) = \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left[-\frac{1}{2\sigma^2} \sum_{r=1}^{M} \sum_{n=1}^{N} (y_{rn} - x_{rn}(\theta))^2\right]. \tag{2}$$

Note that the dependence of the model response upon the model parameters is made explicit: $x_n(\theta)$. Bayes' theorem can be used to relate the desired distribution to the likelihood and priors:

$$p(\theta, \sigma^2 | \mathbf{y}) = \frac{p_L(\mathbf{y}|\theta, \sigma^2) \, p_\pi(\theta, \sigma^2)}{p_D(\mathbf{y})}. \tag{3}$$

The term on the left, $p(\theta, \sigma^2 | \mathbf{y})$, is the desired distribution. To get the individual parameter distributions this expression must be integrated over all other parameters. In what follows, the variance σ^2 is absorbed into the parameter vector θ for convenience.

In order to evaluate Eqn. (3) we use the Markov-Chain Monte-Carlo (MCMC) approach. MCMC is a numerical method for evaluating the posterior probability distributions $p(\theta_i | \mathbf{y})$ [7]. The idea is to generate a stationary Markov Chain for each parameter in θ such that the values in the chain are, in fact, samples from the individual parameter posterior distribution. For the ith parameter, an initial parameter value $\theta_i^{(0)}$ chosen from the prior $p_\pi(\theta_i)$. The next element in the chain is found by generating a trial value $\theta_i^* = \theta_i^{(k)} + q(\theta_i^* | \theta_i^{(k)})$ and computing the ratio:

$$r = \frac{p(\theta_i^* | \mathbf{y}) q(\theta_i^* | \theta_i^{(k-1)})}{p(\theta_i^{(k-1)} | \mathbf{y}) q(\theta_i^{(k-1)} | \theta_i^*)} = \frac{p_L(\mathbf{y}|\theta_i^*) p_\pi(\theta_i^*) \, q(\theta^* | \theta_i^{(k-1)})}{p_L(\mathbf{y}|\theta_i^{k-1}) p_\pi(\theta_i^{k-1}) \, q(\theta_i^{(k-1)} | \theta_i^*)} \tag{4}$$

and accepting the new value with probability $\min(r, 1)$. If the trial succeeds, the perturbed parameter value becomes the next value in the chain: $\theta_i^{(k)} = \theta_i^*$, otherwise the previous value is retained. The process (accept/reject) repeats for each parameter in the vector while holding the other parameters fixed, allowing each parameter to be considered individually. Note that the ratio r removes the need for the distribution $p_D(\mathbf{y})$, since it divides out. Thus the MCMC chain eliminates the need for assessing Bayes' theorem directly. Repeating this process a sufficient number of times generates a chain that forms a probability density function (PDF) for the true value of the crack parameters.

SIMULATION AND RESULTS

The plate under consideration is rectangular and simply supported on all sides. The plate dimensions are given in Table 1. For the sake of this work, the "measured data" $\mathbf{y}$ is a simulation with a crack geometry given in Table 2. 25dB noise was added to this signal to mimic experimental noise. The data in Table 2 is referred to as the "true" parameter set θ.

Four displacement sensors (where the time series were recorded) were used. The sensor locations are given by the triangles in Figure 2a. The initial conditions approximated a hammer strike on the plate starting from rest. The strike locations are shown in Figure 2a as circles. The Bernoulli trial success rate varied between 35% and 50%. A total of seven runs were made, each with a different crack length (5cm $< a <$ 11cm); the point was to see how well the technique identified successively smaller cracks.

Dimension	Value
Length	1.25 m
Width	1.00 m
Thickness	0.01 m

Table 1: Plate Dimensions

Dimension	Value
Crack Center	(0.8, 0.6) m
1/2 Crack Length	0.05 to 0.11 m
Crack Angle	-30°

Table 2: Crack Parameters
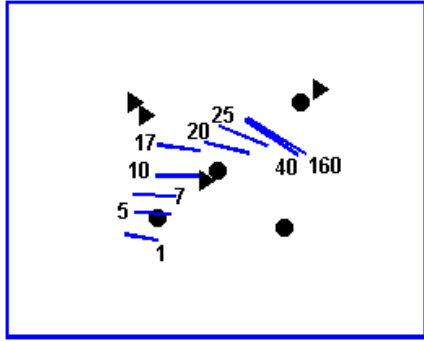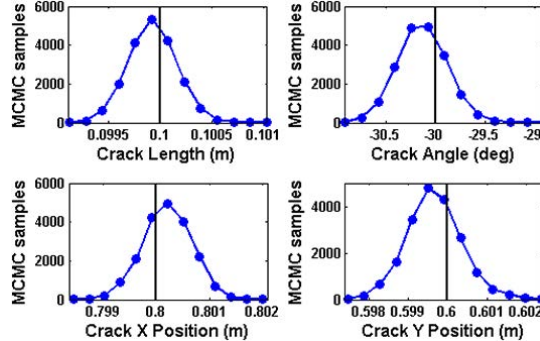
Figure 2: (a) movement of the crack parameter estimates during the burn-in process and (b) the final distributions of the parameters, from the post-burn-in phase of the Markov Chain.

Figure 2a shows how the initial guess (labeled 1) gradually evolves during the "burn-in" phase, for the case $a = 0.1m$. By iteration 160, the estimate is obviously close to the actual answer. After a 1500 iteration burn-in, the chain is extended with an additional 20,000 iterations, which are used to generate the final distributions showing in 2b. These distributions correctly identify the crack parameters with a very tight variance.

Figure 3 shows how the 99% credible intervals vary as a function of crack size. These results show that the proposed approach appears to work well, producing narrow error bars for all parameters. In addition, these results show that the ability to estimate crack parameters is not a monotonic function of crack size. For the crack sizes considered in this study, smaller cracks are not necessarily more difficult to identify than larger ones. However, for still smaller cracks ($< 0.05m$) we expect algorithm performance to degrade.

CONCLUSIONS

In this paper, preliminary results are shown for the viability of using a Bayesian approach to identify the size, location and orientation of a crack in a simply supported plate. The technique shows promise by precisely and accurately locating a crack in the presence of 25dB Gaussian noise. In addition, the crack size does not strongly affect the precision of the parameter estimates.
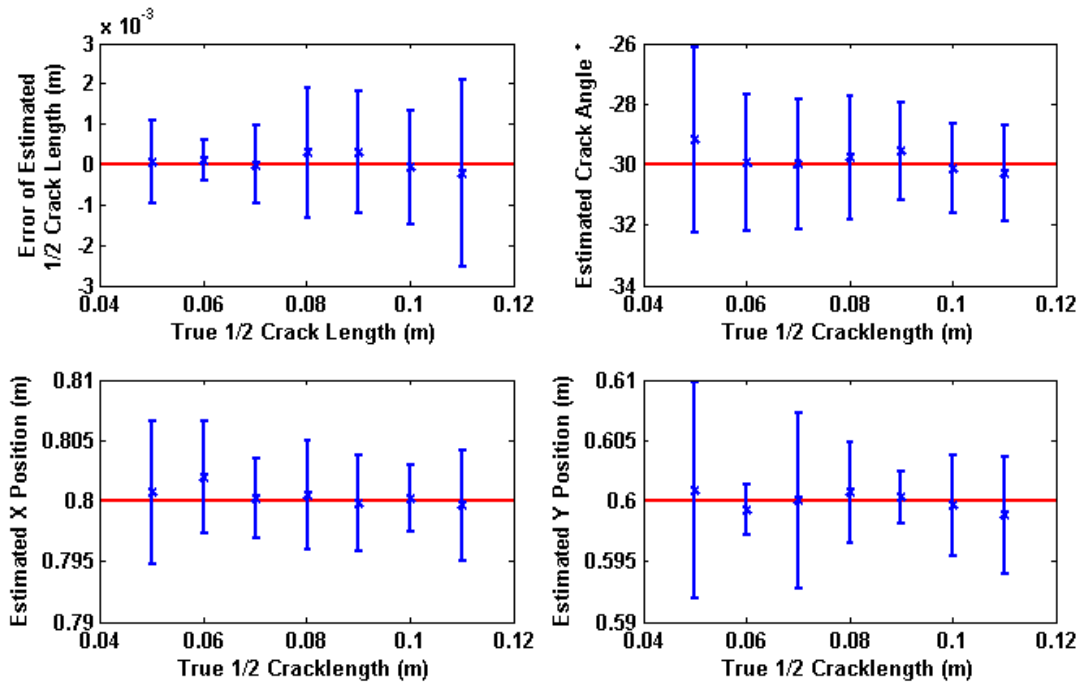
Figure 3: Estimate of crack parameters versus crack length. Error bars represent the 99% credible interval. The first figure shows the *error* of the crack length.

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